

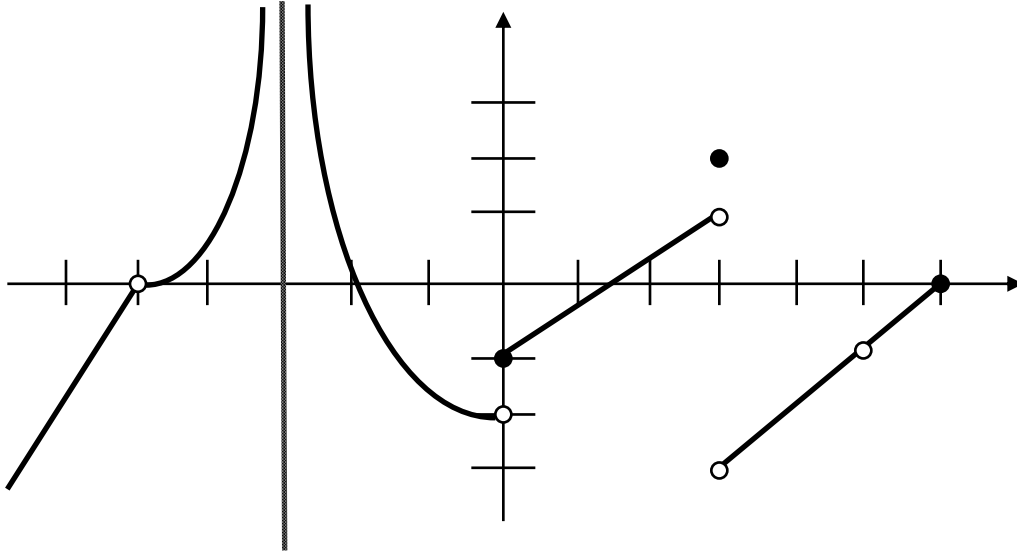
Name _____

AP Calculus BC

Summer Review Packet (Limits & Derivatives)

Limits

1. Answer the following questions using the graph of $f(x)$ given below.



(a) Find $f(0)$

(b) Find $f(3)$

(c) Find $\lim_{x \rightarrow -5} f(x)$

(d) Find $\lim_{x \rightarrow 0^+} f(x)$

(e) Find $\lim_{x \rightarrow 3^-} f(x)$

(f) Find $\lim_{x \rightarrow -3^+} f(x)$

(g) List all x -values for which $f(x)$ has a removable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

- (h) List all x -values for which $f(x)$ has a nonremovable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

In problems 2-10, find the limit (if it exists) using analytic methods (i.e. without using a calculator).

2.
$$\lim_{x \rightarrow -2} \frac{3x^2 + 21x + 30}{x^3 + 8}$$

3.
$$\lim_{x \rightarrow \pi/6} \frac{1 - \cos^2 x}{4x}$$

4.
$$\lim_{x \rightarrow 4} \frac{\sqrt{x-3} - 1}{x-4}$$

5.
$$\lim_{x \rightarrow 0} \frac{[1/(x+1)] - 1}{x}$$

6.
$$\lim_{x \rightarrow 0} \frac{\left[1/\sqrt{1+x}\right] - 1}{x}$$

7.
$$\lim_{\theta \rightarrow 0} \frac{\sin 6\theta^3}{7\theta}$$

8.
$$\lim_{t \rightarrow 0} \frac{\sin^2 3t^2}{t^3}$$

9.
$$\lim_{x \rightarrow 6^-} \frac{|6x - 36|}{6 - x}$$

10.
$$\lim_{\Delta x \rightarrow 0} \frac{\sin((\pi/6) + \Delta x) - (1/2)}{\Delta x}$$

Hint: $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

11. Suppose $f(x) = \begin{cases} \frac{\sqrt{2x+1}-\sqrt{3}}{x-1}, & x \geq 0 \\ 4x^2 + k, & x < 0 \end{cases}$.

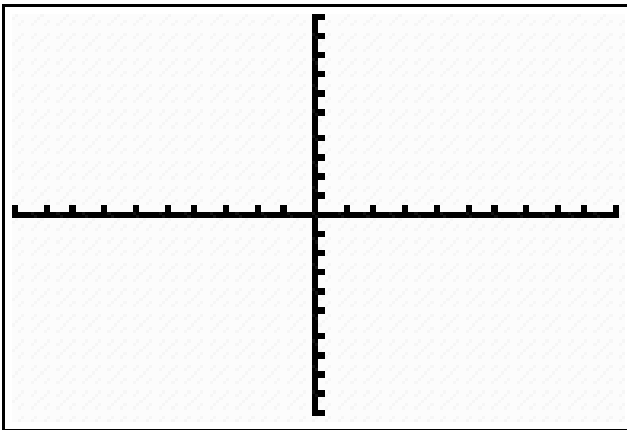
(a) For what value of k will f be piecewise continuous at $x = 0$? Explain why this is true using one-sided limits. (**Hint: A function is continuous at**

$x = c$ if (1) $f(c)$ exists, (2) $\lim_{x \rightarrow c} f(x)$ exists, and (3) $\lim_{x \rightarrow c} f(x) = f(c)$.)

(b) Using the value of k that you found in part (a), **accurately** graph f below.

Approximate the value of $\lim_{x \rightarrow 1} f(x)$

$\lim_{x \rightarrow 1} f(x) =$ _____



(c) Rationalize the numerator to find $\lim_{x \rightarrow 1} f(x)$ analytically.

12. **Analytically** determine the values of b and c such that the function f is continuous on the entire real number line. **See the hint given in problem 11.**

$$f(x) = \begin{cases} x+1, & 1 < x < 3 \\ x^2 + bx + c, & x < 1 \text{ or } x > 3 \end{cases}$$

In problem 13, circle the correct answer and explain why the answer is the correct one.

13. If $f(x) = x^3 + x - 3$, and if c is the only real number such that $f(c) = 0$, then by the Intermediate Value Theorem, c is necessarily between

- (A) -2 and -1
- (B) -1 and 0
- (C) 0 and 1
- (D) 1 and 2
- (E) 2 and 3

Hint: The Intermediate Value Theorem states that if f is a continuous function on the interval $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there must exist at least one number $c \in [a, b]$ such that $f(c) = k$.

Derivatives

In problems 1 & 2, find the derivative of the function by using the limit definition of the derivative.

1. $f(x) = x^3 - 2x + 3$

2. $f(x) = \frac{x+1}{x-1}$

In problems 3-14, find the derivative of the given function using the power, product, quotient, and/or chain rules.

3. $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

4. $f(x) = \sqrt{x} \sin x$

5. $f(t) = t^3 \cos t$

6. $f(x) = \frac{x^2 + x - 1}{x^2 - 1}$

7. $f(x) = \frac{x^4 + x}{\tan^2 x}$

8. $f(x) = 3x^2 \sec^3 x$

9. $f(x) = 3x \csc x + x \cot x$

10. $f(x) = \left(\frac{x+5}{x^2-6x} \right)^2$

11. $f(x) = (x^3 - 2)^{3/2} (5x^2 + 1)^{5/2}$

12. $f(x) = x^3 \cot^4(7x)$

13. $f(x) = 5 \sin^2(\sqrt{3x^4 + 1})$

Problems continue on the next page.

In problems 14 & 15, find an equation of the tangent line to the graph of f at the indicated point P .

14. $f(x) = \frac{1 + \cos x}{1 - \cos x}, P\left(\frac{\pi}{2}, 1\right)$

15. $f(x) = (x^2 - 1)^{2/3}, P(3, 4)$

In problems 16 & 17, find the second derivative of the given function.

16. $f(x) = (4x^2 - 3x)^{3/2}$

17. $h(x) = x^3 \cos(\pi x)$

In problem 18, use the position function $s(t) = -16t^2 + v_0t + s_0$ for free-falling objects.

18. A ball is thrown straight down from the top of a 220-foot tall building with an initial velocity of -22 feet per second.

(a) Determine the average velocity of the ball on the interval $[1, 2]$.

(b) Determine the instantaneous velocity of the ball at $t = 3$.

(c) Determine the time t at which the average velocity on $[0, 2]$ equals the instantaneous velocity.

(d) What is the velocity of the ball when it strikes the ground?

In problem 19-24, circle the correct answer and explain why the answer is the correct one.

19.
$$\lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{6} + h\right) - \cos\left(\frac{\pi}{6}\right)}{h} =$$

(A) Does not exist

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) $\frac{\sqrt{3}}{2}$

(E) $-\frac{\sqrt{3}}{2}$

20. Let f and g be differentiable functions with values for $f(x)$, $g(x)$, $f'(x)$, and $g'(x)$ shown below for $x = 1$ and $x = 2$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	-4	12	-8
2	5	1	-6	4

Find the value of the derivative of $f(x) \cdot g(x)$ at $x = 1$.

- (A) -96
(B) -80
(C) -48
(D) -32
(E) 0
21. Let $f(x) = \begin{cases} 3x^2 + 4, & x < 1 \\ x^3 + 3x, & x \geq 1 \end{cases}$. Which of the following is true?
- I. $f(x)$ is continuous at $x = 1$
II. $f(x)$ is differentiable at $x = 1$
III. $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$
- (A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

22. The equation of the line tangent to the curve $f(x) = \frac{kx+8}{k+x}$ at $x = -2$ is $y = x + 4$. What is the value of k ?

(A) -3

(B) -1

(C) 1

(D) 3

(E) 4

23. An equation of the line normal to the curve $y = \sqrt[3]{x^2 - 1}$ at the point where $x = 3$ is

(A) $y + 12x = 38$

(B) $y - 4x = 10$

(C) $y + 2x = 4$

(D) $y + 2x = 8$

(E) $y - 2x = -4$

Hint: A normal line to a curve at a point is perpendicular to the tangent line to the curve at the same point.

24. If the n th derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(14)}$ is the same as

(A) y

(B) $\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2}$

(D) $\frac{d^3y}{dx^3}$

(E) None of the above

Answers

Limits:

1. (a) -1

(b) 2

(c) 0

(d) -1

(e) 1

(f) $+\infty$

(g) $x = -5, 5$

(h) $x = -3, 0, 3$

2. $3/4$

3. $3/(8\pi)$

4. $1/2$

5. -1

6. $-1/2$

7. 0

8. 0

9. 6

10. $\frac{\sqrt{3}}{2}$

11. (a) $k = -1 + \sqrt{3}$

(b) $\approx .577$

(c) $\frac{1}{\sqrt{3}}$

12. $b = -3, c = 4$

13. D

Derivatives:

1. $f'(x) = 3x^2 - 2$

2. $f'(x) = \frac{-2}{(x-1)^2}$

3. $f'(x) = 12x^3 - 18x^2 + 32x - 14$

4. $f'(x) = \sqrt{x} \cos x + \frac{\sin x}{2\sqrt{x}}$

5. $f'(t) = -t^3 \sin t + 3t^2 \cos t$

6. $f'(x) = \frac{-x^2 - 1}{(x^2 - 1)^2}$

7. $f'(x) = \frac{4x^3 \tan x + \tan x - 2x^4 \sec^2 x - 2x \sec^2 x}{\tan^3 x}$

8. $f'(x) = 9x^2 \sec^3 x \tan x + 6x \sec^3 x$

9. $f'(x) = -3x \csc x \cot x + 3 \csc x - x \csc^2 x + \cot x$

10. $f'(x) = \frac{(2x+10)(-x^2-10x+30)}{(x^2-6x)^3}$

11. $f'(x) = 25x((x^3-2)(5x^2+1))^{3/2} + \frac{9}{2}x^2(5x^2+1)^{5/2}(x^3-2)^{1/2}$

12. $f'(x) = -28x^3 \cot^3(7x) \csc^2(7x) + 3x^2 \cot^4(7x)$

13. $f'(x) = \frac{60x^3 \sin \sqrt{3x^4+1} \cos \sqrt{3x^4+1}}{\sqrt{3x^4+1}}$

14. $y-1 = -2\left(x - \frac{\pi}{2}\right)$

15. $y-4 = 2(x-3)$

16. $f'(x) = 12\sqrt{4x^3-3x} + \frac{3(8x-3)^2}{4\sqrt{4x^3-3x}}$

17. $h'(x) = -\pi^2 x^3 \cos \pi x - 6\pi x^2 \sin \pi x + 6x \cos \pi x$

18. (a) **-70 ft./s.**

(b) **-118 ft./s.**

(c) **$t = 1$ s.**

(d) **≈ -120.688 ft./s.**

19. **C**

20. **B**

21. **B**

22. **D**

23. **D**

24. **C**